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Postbuckling of Thermally Stressed Composite Plates

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I. Introduction

STRUCTURES made of composite materials often operate at elevated temperatures. At these temperatures thermomechanical stressing may occur. Composite structures that are thermally stressed may buckle and postbuckle from subsequent application of elastic loads. It is, therefore, of interest to examine the postbuckling behavior of structures made of composite materials that are thermally stressed. Of particular interest is the effect of initial thermal stressing to the subsequent postbuckling behavior of composite panels. The present study considers two laminated composite plates, namely, an eight-layer $(0/90/0/90)_s$ cross-ply laminate and a eight-layer quasi-isotropic $(45/-45/0/90)_s$ composite plate. Both plates are thermally stressed via the application of various temperatures. Following thermal stressing an axial load is incrementally applied, and the postbuckling behavior is examined.

Some analytical and numerical studies concerning the behavior of composite structures at elevated temperatures have been reported.[1–5] Most of these studies, however, are concerned solely with thermal buckling and postbuckling of composite plates. The present study considers the laminated plates to be in a state of thermal stressing before application of the elastic compressive loads. The effect of initial thermal stressing to the final postbuckling behavior is assessed.

II. Computational Experiments

Figure 1 shows a laminated composite plate along with all geometrical and material properties. The left edge of the plate cannot move in the three directions, whereas for all other edges vertical

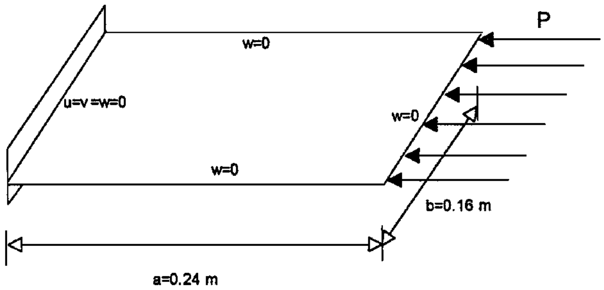

Fig. 1 Laminated composite plate; geometrical and material data: $E_1 = 150$ GPa, $E_2 = E_3 = 10$ GPa, $G_{12} = G_{13} = 6$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $a_{t1} = 2.5 \times 10^{-8}$ °C^{-1}, $a_{t2} = 30 \times 10^{-6}$ °C^{-1}, and $h_L = 3.125 \times 10^{-4}$ m.

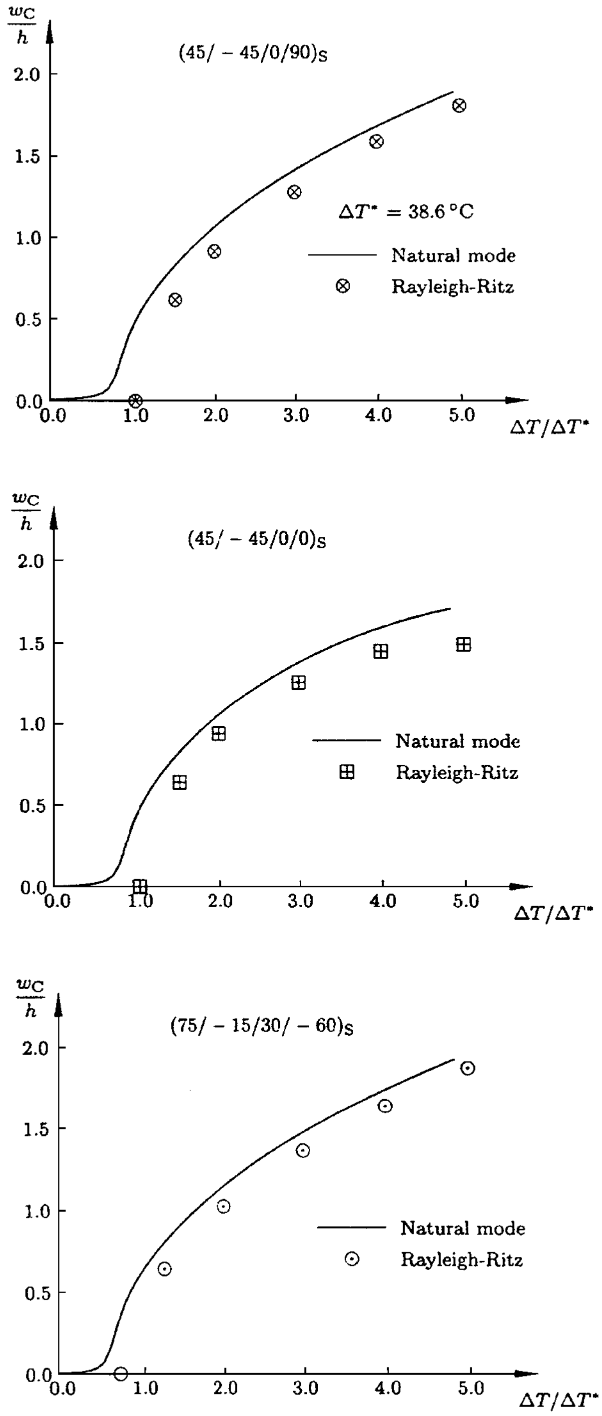

Fig. 2 Comparison of present natural-mode method with reported results [Rayleigh–Ritz method (Ref. 4)] for thermal postbuckling of three composite plates.

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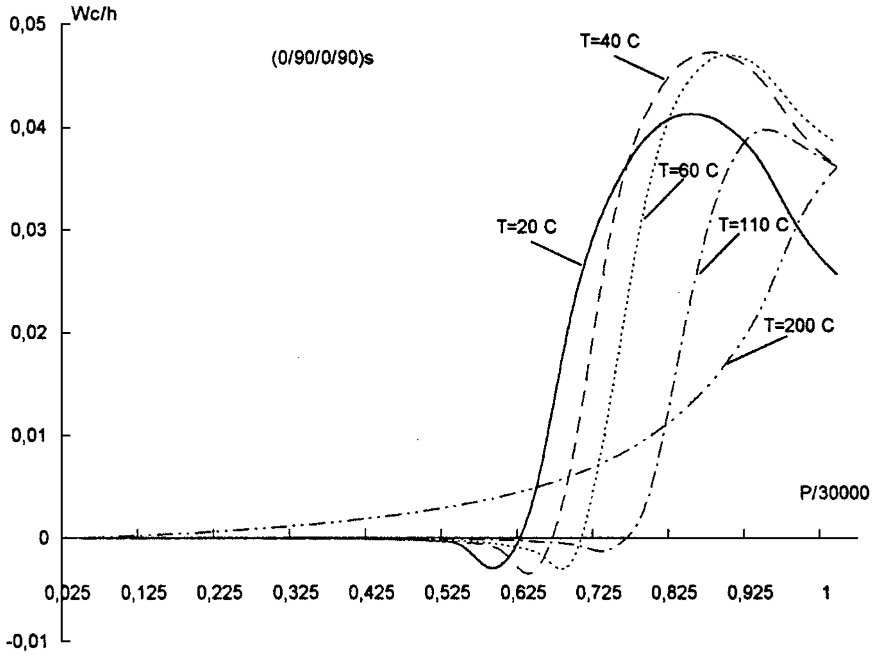

Fig. 3 Normalized load-central displacement curves for the $(0/90/0/90)_s$ laminate for various temperatures (normalized load units are in newton).

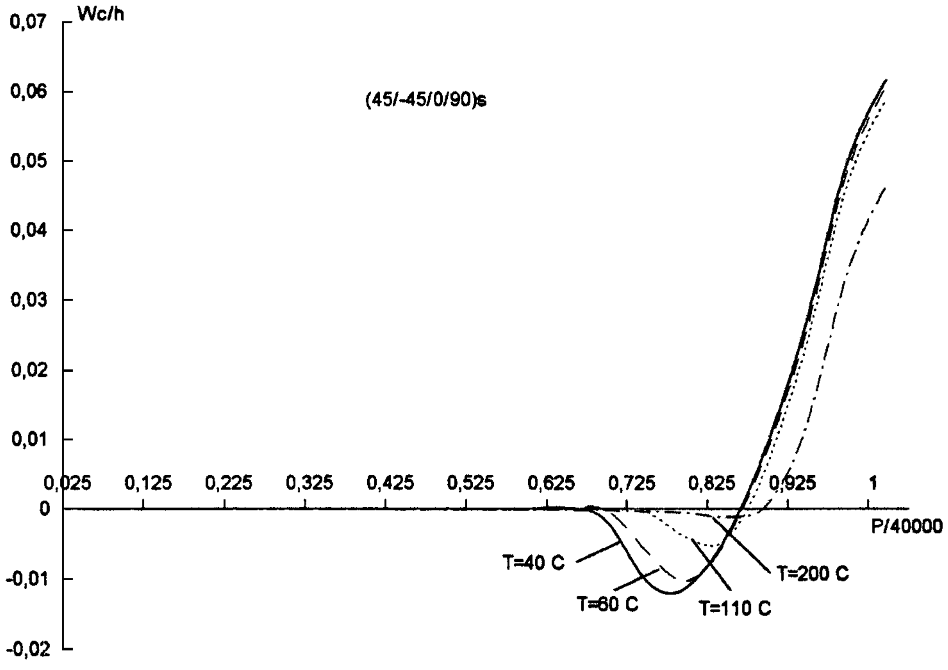

Fig. 4 Normalized load-central displacement curves for the $(45/-45/0/90)_s$ laminate for various temperatures (normalized load units are in newton).

displacement is prohibited. Uniform temperature increase is applied on the top and bottom boundaries. An axial compressive load is imposed on the right plate edge. Two eight-layer laminations are considered, namely, cross-ply $(0/90/0/90)_s$, and quasi isotropic $(45/-45/0/90)_s$. The geometrically nonlinear problem in hand is solved in two stages. First, the temperature is applied, and the following nonlinear problem is considered:

$$\boldsymbol{K}_T \boldsymbol{r} = \boldsymbol{J} \tag{1}$$

where $\boldsymbol{K}_T$ is the tangent stiffness and $\boldsymbol{J}$ is the initial load due to temperature. Following convergence and the full application of the incremental temperature, a second geometrically nonlinear problem is formed, namely,

$$\boldsymbol{K}_T \boldsymbol{r} = \boldsymbol{R} \tag{2}$$

where $\boldsymbol{R}$ is the global vector of the applied axial loads and $\boldsymbol{r}$ the global unknown vector of the nodal displacements and rotations.

The composite plate is discretized with a set of triangular elements based on the natural-mode finite element method.[5,6] In brief, the triangular finite element is assigned a set of rigid-body and straining modes. The latter are equal to the global nodal degrees of freedom minus the number of rigid-body modes. The geometrically nonlinear problem is solved incrementally in two stages: first for the applied temperature and subsequently for the applied axial external compressive edge loads.

Figure 2 shows the comparison of the present element and methodology[5] (natural-mode method) with results of the literature [Rayleig–Ritz method (Ref. 4)] for thermal postbuckling of composite plates. Shown therein is the thermal postbuckling behavior of three laminates, namely, $(45/-45/0/90)_s$, $(45/-45/0/0)_s$, and $(75/-15/30/-60)_s$ composite plates. The comparison of the present natural-mode method with the reported Rayleigh–Ritz method is very good.

Figure 3 presents the load-displacement curves for a central laminate point for the $(0/90/0/90)_s$ composite plate for different applied temperatures. The temperature range is between $T = 20$ and 200°C. All material properties are assumed independent of temperature. We observe that initially the central point displaces on the negative side and subsequently it moves upward. An important observation is that an increase of temperature and thermal stressing increases the buckling load. However, at $T = 200$°C, the cross-ply laminate shows a completely different behavior. Figure 4 presents the load-displacement curves for the quasi-isotropic $(45/-45/0/90)_s$ laminate. Unlike the cross-ply $(0/90/0/90)_s$ laminate, the postbuckling behavior of the quasi-isotropic laminate is not very sensitive to the application of various temperatures and initial thermal stressing. Also, the buckling load of this laminate is significantly higher than the cross-ply composite plate. Only at $T = 200$°C, the load-displacement curve moves somewhat to the right. We, therefore, conclude that the postbuckling behavior of the $(45/-45/0/90)_s$ laminate is less sensitive to initial thermal stressing than the corresponding behavior of the cross-ply $(0/90/0/90)_s$ composite plate.

III. Conclusions

The postbuckling behavior of two eight-layer laminated composite plates is considered, namely, a cross-ply $(0/90/0/90)_s$ and a quasi-isotropic $(45/-45/0/90)_s$ laminate. Initially, both plates are thermally stressed via the application of various temperatures. Then, an axial compressive load is applied, and the laminates' postbuckling behavior is considered. The $(0/90/0/90)_s$ laminate is very sensitive to the amount of initial thermal stressing, whereas the $(45/-45/0/90)_s$ plate exhibits less sensitivity to initial thermal stressing. Normalized load-central-displacement curves are provided for both laminates for various temperatures.

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